# SAT MATH FORMULA SHEET

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## NUMBERS & OPERATIONS

### EVEN & ODD

Even ± Even = Even  Even ± Odd = Odd

Odd ± Odd = Even

Even · Anything = Even  Odd · Odd = Odd

The integer zero (0) is an even number.

### DIVISIBILITY

| A number is divisible by... | if... |
|---|---|
| **2** | it is even or the ones digit is 0, 2, 4, 6, 8. |
| **3** | the sum of the digits is divisible by 3. |
| **4** | the number taken from the last two digits is divisible by 4 or it is divisible by 2 twice. |
| **5** | the ones digit is 0 or 5. |
| **6** | it is even and divisible by 3. |
| **9** | the sum of its digit is divisible by 9. |

### PRIME NUMBER

A prime number is a positive integer which has exactly two distinct divisors: 1 and itself. That means, by definition, 1 is not a prime number. 2 is the smallest and the only even prime number. Prime numbers less than 20: 2, 3, 5, 7, 11, 13, 17, 19

### ORDER OF OPERATIONS

**PEMDAS**
1. **P**arentheses.
2. **E**xponents.
3. **M**ultiplication and **D**ivision from left to right.
4. **A**ddition and **S**ubtraction from left to right.

**Ex:** $3\times(5+8)-2^2\div4+3$
$= 3\times13-2^2\div4+3$
$= 3\times13-4\div4+3$
$= 39-1+3 = 38+3 = 41$

### FRACTIONS

**Simplifying Fractions**
If a fraction is in its simplest form, then the numerator and the denominator contain no common factor other than 1. Otherwise, simplify the fraction by dividing both the numerator and the denominator by the common factors until the only common factor that is left is 1. When working with fractions on the SAT, always simplify first.

**Ex:** $\frac{18}{60} = \frac{9}{30} = \frac{3}{10}$

**Adding and Subtracting Fractions**
1. Find the least common denominator (LCD) of the two fractions.
2. Rewrite the fractions as equivalent fractions with the LCD as the denominator.
3. Add or subtract the numerators.

**Ex:** $\frac{3}{8}+\frac{5}{12} = \frac{3\times3}{8\times3}+\frac{5\times2}{12\times2} = \frac{9}{24}+\frac{10}{24} = \frac{9+10}{24} = \frac{19}{24}$

**Multiplying Fractions**
1. Cross cancel any common factors from the denominators and the numerators.
2. Multiply the numerators and multiply the denominators.

**Ex:** $\frac{\cancel{5}^{1}}{\cancel{6}_{2}}\times\frac{\cancel{3}^{1}}{\cancel{10}_{2}} = \frac{1\times1}{2\times2} = \frac{1}{4}$

**Dividing Fractions**
To divide any number by a fraction, multiply the number the reciprocal of the divisor fraction.

**Ex:** $\frac{3}{5}\div\frac{2}{7} = \frac{3}{5}\times\frac{7}{2} = \frac{3\times7}{5\times2} = \frac{21}{10}$

**Multiplying/Dividing Fractions with Decimals**
Convert decimals into fractions, and then calculate.

**Ex:** $0.33\times\frac{2}{11}\div0.6 = \frac{33}{100}\times\frac{2}{11}\div\frac{3}{5} = \frac{\cancel{\cancel{33}}^{1}}{\cancel{\cancel{100}}_{10}}\times\frac{\cancel{2}^{1}}{\cancel{11}_{1}}\times\frac{\cancel{5}^{1}}{\cancel{3}_{1}} = \frac{1}{10}$

### RADICALS

**Simplifying Radicals**
Move square factors out of the radical until no square factor is left.

**Ex:** $\sqrt{800} = \sqrt{8\times100} = \sqrt{8}\times\sqrt{100} = 10\sqrt{8} = 10\sqrt{4\times2}$
$= 10\times\sqrt{4}\times\sqrt{2} = 10\times2\times\sqrt{2} = 20\sqrt{2}$

Similar to simplifying fractions, simplify the radicals first when working with them on the SAT.

**Multiplying Radicals**
Multiply the numbers under the radicals, then write the product under a single radical.

**Ex:** $\sqrt{3}\times\sqrt{2} = \sqrt{3\times2} = \sqrt{6}$

**Dividing Radicals**
Similar to multiplying radicals, divide the numbers under the radicals. Write the quotient under a single radical.

**Ex:** $\frac{\sqrt{42}}{\sqrt{30}} = \sqrt{\frac{42}{30}} = \sqrt{\frac{7}{5}}$

**Rationalizing the Denominator**
Multiply the numerator and the denominator by the radical in the denominator.

**Ex:** $\frac{1}{\sqrt{2}} = \frac{1\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2^2}} = \frac{\sqrt{2}}{2}$

### SEQUENCES

## ALGEBRA & FUNCTIONS

### EXPONENTS

### COMMON PRODUCTS

**FOIL Method**
When multiplying binomials, expressions that contain exactly two terms, apply the **FOIL** (**F**irst **O**uter **I**nner **L**ast) method.

**Ex:** $(x+1)(2x+3) = 2x^2+3x+2x+3 = 2x^2+5x+3$

**Perfect Squares**

$(x+y)^2 = x^2+2xy+y^2$

**Ex:** $(5x+2y)^2 = (5x)^2+2\cdot5x\cdot2y+(2y)^2$
$= 25x^2+20xy+4y^2$

$(x-y)^2 = x^2-2xy+y^2$

**Ex:** $(\sqrt{5}-\sqrt{3})^2 = (\sqrt{5})^2-2\cdot\sqrt{5}\cdot\sqrt{3}+(\sqrt{3})^2 = 5-2\sqrt{15}+3$
$= 8-2\sqrt{15}$

**Difference of Squares**
$(x+y)(x-y) = x^2-y^2$

**Ex:** $(3-\sqrt{5})(3+\sqrt{5}) = 3^2-(\sqrt{5})^2 = 9-5 = 4$

### SOLVING EQUATIONS

# ALGEBRA & FUNCTIONS (CONTINUED)

## UNSOLVABLE VARIABLES

## ABSOLUTE VALUES

The absolute value of $x$, denoted by $|x|$, is the distance of $x$ from 0. It is always nonnegative. Formally,

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

**Ex:** $|2-8| = |-6| = 6$

If $|x| = y$, then $x = y$ or $x = -y$.

**Ex:** Solve for $x$.

$$|3x+1| = 5$$

$$3x+1 = 5 \text{ or } 3x+1 = -5$$

$$x = \frac{4}{3} \text{ or } x = -2$$

## SIMULTANEOUS LINEAR EQUATIONS

**Solving by Substitution**

1. From any of the two equations, write $y$ in terms of $x$.
2. Plug the expression for $y$ into the other equation.
3. Solve for $x$ in the new one-variable linear equation.
4. Compute $y$ by plugging the value of $x$ into the expression found in step 2.

For what value of $x$ and $y$ are the following equations both true?

**Ex:** $\begin{cases} x+y = 3 & (1) \\ x-y = 1 & (2) \end{cases}$

Using equation (1) to write $y$ in terms of $x$ gives $y = 3-x$. Plugging into equation (2) gives $x-(3-x) = 1$. Solving $x$ yields $x = 2$. Plugging in the expression for $y$ gives $y = 1$.

**Solving by Adding or Subtracting Equations**

The goal is to form a new one-variable equation by adding or subtracting the two original equations.

- If the coefficients of a variable are the same in the two equations, subtract the two equations.
- If the coefficients of a variable are of different signs in the two equations, add the two equations.
- Otherwise, match the coefficients of one variable in both equations by multiplying the proper factors, then add or subtract the two equations.

The new equation will only involve one variable. Solve it. Plug the solution back into any one of the original equations to solve for the other variable.

## INEQUALITIES

## QUADRATIC EQUATIONS

## WORD PROBLEMS

## PERCENTS, RATIOS, AND PROPORTIONS

## RATE PROBLEMS

**Equations**

| | |
|---|---|
| Distance = Rate · Time | Work = (Work Rate) · Time |
| Time = $\frac{\text{Distance}}{\text{Rate}}$ | Time = $\frac{\text{Work}}{\text{Work Rate}}$ |
| Rate = $\frac{\text{Distance}}{\text{Time}}$ | Work Rate = $\frac{\text{Work}}{\text{Time}}$ |

**Ex:** A bus traveling at an average rate of $x$ miles per hour made the trip to the city in 8 hours. If it had traveled at an average rate of $3/4x$ miles per hour, how many more hours would it have taken to make the same trip?

Let $D$ denote the distance of the trip, then $D = \text{Rate} \times \text{Time} = 8x$

Let $T_2$ denote the time it takes the bus to travel the same trip at an average rate of $3/4x$ miles per hour.

$$T_2 = \frac{\text{Distance}}{\text{Rate}} = \frac{8x}{\frac{3}{4}x} = \frac{32}{3}$$

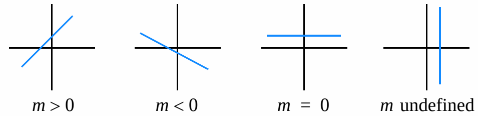
Taking the difference yields $T_2 - 8 = \frac{32}{3} - 8 = \frac{8}{3}$.

# ALGEBRA & FUNCTIONS (CONTINUED)

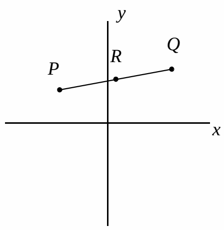
## FUNCTIONS



**Rough Directions of the Slopes**




**Distance and Midpoint Formulas**

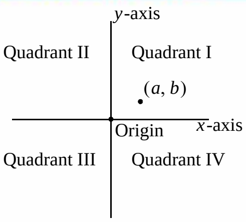



The coordinates of P and Q are $(x_1, y_1)$ and $(x_2, y_2)$ respectively. The distance between P and Q is:

$$\overline{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

If R is the midpoint of line segment PQ, then R's coordinates are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

## COORDINATES




The line $x = 0$ lies on the $y$-axis. The line $y = 0$ lies on the $x$-axis.

**Equation of a Line**

Equation: $y = mx + b$

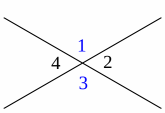
where $m$ is the slope and $b$ is the $y$-intercept. We will be using this representation throughout the coordinate section.

# GEOMETRY

## ANGLES

**Vertical Angles**

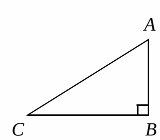
**Vertical angles** are the opposite angles formed by two intersecting lines. Vertical angles have the same measure.




$m\angle 1 = m\angle 3, m\angle 2 = m\angle 4$

**Complementary Angles**

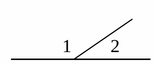
Two angles whose measures have a sum of $90°$ are called **complementary angles**.




In right triangle $ABC$, $\angle ACB$ and $\angle CAB$ are complementary angles because $m\angle ACB + m\angle CAB = 180° - 90° = 90°$.

**Supplementary Angles**

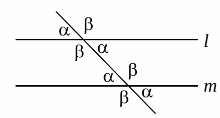
Two angles whose measures sum to 180 are call **supplementary angles**, i.e. two angles formed by a straight line.




$m\angle 1 + m\angle 2 = 180°$

**Parallel Lines**

Two lines that do not intersect or meet are called **parallel lines**.

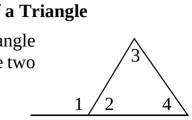



In the figure, lines $l$ and $m$ are parallel, written as $l \parallel m$. Angles with the same name have the same measure. Also $\angle\alpha + \angle\beta = 180°$.

**Exterior & Remote Interior Angles of a Triangle**

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.

$m\angle 1 = m\angle 3 + m\angle 4$



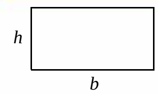
# GEOMETRY (CONTINUED)

## TRIANGLES

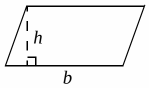
## POLYGONS

## CIRCLES

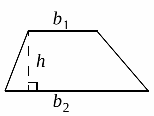
## MEASUREMENTS



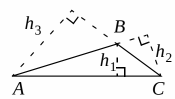

**Rectangle**
Area = base · height = $bh$
Perimeter = 2(base + height) = $2(b + h)$



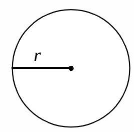

**Parallelogram**
Area = base · height = $bh$



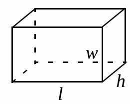

**Trapezoid**
Area=(average base)height = $\frac{1}{2}(b_1 + b_2)h$



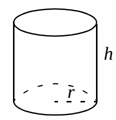

**Triangle**
Area = $\frac{1}{2}$base · height
$= \frac{1}{2}AC \cdot h_1 = \frac{1}{2}AB \cdot h_2 = \frac{1}{2}BC \cdot h_3$




**Circle**
Area = $\pi \cdot (\text{radius})^2 = \pi r^2$
Perimeter = $2\pi \cdot$ radius = $2\pi r$




**Rectangular Solid**
Surface Area = $2(lw + lh + hw)$
Volume = (Base area) · height = $lwh$




**Cylinder**
Surface Area = $2\pi r^2 + 2\pi rh$
Volume = (Base area) · height = $\pi r^2 h$

# STATISTICS, COUNTING, AND PROBABILITY

## STATISTICS

**Arithmetic Mean/Average**
The **arithmetic mean** of a list of $n$ values is defined as the sum of the $n$ values divided by $n$.
The arithmetic mean of a list of $n$ evenly spaced values is equal to the sum of the smallest value and the largest value divided by 2.

**Ex:** The arithmetic mean of the set {3, 6, 9, 12, 15} is
$$\frac{3+6+9+12+15}{5} = 9 \text{ or } \frac{3+15}{2} = 9.$$

**Median**
If a list of $n$ values are ordered from least to greatest, the **median** is defined as the middle value if $n$ is odd and the sum of the two middle values divided by 2 if $n$ is even.
The median is equal to the arithmetic mean for a list of $n$ evenly spaced values.

**Ex:** The median of the set {4, 1, 6, 9} is $\frac{4+6}{2} = 5$.

**Mode**
The **mode** of a list of values is the value or values that appear the greatest number of times.
**Ex:** The mode of the set {5, 6, 1, 5, 6, 2, 6} is 6.

**Multiple Modes**
**Ex:** Consider the following list:
2, 3, 5, 2, 2, 6, 5, 5, 100, 12, 6, 3, 3
In the list above, there are three modes: 2, 3, and 5.

**Range**
The **range** of a list of values is defined as the greatest value minus the least value.
**Ex:** The range of the set {2, 6, 4, 1, 5, 6, 3} is 6 – 1 = 5.

You do **NOT** need to know the computation of standard deviation for the SAT.

## COUNTING

## PROBABILITY